UCRHEP-T315
September 2001

Double Threefold Degeneracies for Active and Sterile Neutrinos

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Abstract

We explore the possibility that the 3 active (doublet) neutrinos have nearly degenerate masses which are split only by the usual seesaw mechanism from 3 sterile (singlet) neutrinos in the presence of a softly broken A_4 symmetry. We take the unconventional view that the sterile neutrinos may be light, i.e. less than 1 keV, and discuss some very interesting and novel phenomenology, including a connection between the LSND neutrino data and solar neutrino oscillations.

Present experimental data [1, 2, 3] indicate that neutrinos oscillate. Hence they should have small nonzero masses and mix with one another. On the other hand, since neutrino oscillations only measure the difference of mass squares, the possibility that all 3 active neutrinos are nearly degenerate in mass should not be overlooked [4]. There are two canonical ways of making m_ν nonzero. One is through the small vacuum expectation value (VEV) of a Higgs triplet [5, 6]. The other is through the addition of 3 heavy singlet neutral fermions (usually considered as right-handed neutrinos N_R). In that case, a Dirac mass m_D linking the left-handed doublet neutrinos ν_L with N_R as well as a Majorana mass M for N_R are allowed. Combining the two mechanisms, the following mass matrix

$$\mathcal{M}_{\nu N} = \begin{pmatrix} m_0 & m_D \\ m_D & M \end{pmatrix} \tag{1}$$

is obtained. The eigenvalues are simply $m_0 - m_D^2/M$ and M. Without m_0 (which comes from the VEV of the Higgs triplet), this is just the famous seesaw mechanism [7] for a small neutrino mass. The singlet N_R is too heavy to be detected experimentally, unless [8] m_D comes from a different Higgs doublet with a suppressed VEV, in which case M may in fact be only a few TeV or less and become observable at future colliders. Using the model of Ref.[8], it has also been shown [9] that the possibly large observed discrepancy of the muon anomalous magnetic moment [10] may be explained, provided that the 3 active neutrinos are in fact nearly degenerate in mass, in order not to conflict with the present experimental bound on $\tau \to \mu\gamma$.

In this paper we consider the case where both m_D and M are small, but m_D is still less than M by perhaps an order of magnitude. This is in contrast to the pseudo-Dirac scenario [11], i.e. $m_0, M << m_D$, in which case neutrino oscillations would be maximal between active and sterile species, in disfavor with the most recent data [1, 2]. We also supplement our model with a discrete A_4 symmetry [12, 13] which maintains the separate degeneracies of the 3 active and 3 sterile neutrinos. This A_4 is then broken spontaneously and softly to allow for realistic charged-lepton masses as well as neutrino mass differences as in Ref.[12]. The new idea here is that the 3 sterile neutrinos could be light and help to account for the LSND data [3] as shown below.

Before discussing the theoretical reasons for m_0, m_D, and M to be small, consider first the phenomenology of such a possibility. The 3 active neutrinos ν_e, ν_μ, ν_τ are now each a linear combination of 6 light neutrino mass eigenstates. With m_D less than M by an order of magnitude, the mixing of N with ν is still small; hence the presumably large mixings among the 3 active neutrinos themselves are sufficient to explain the atmospheric [1] and solar [2] neutrino data. This leaves the LSND data [3] to be explained by the mixing of ν with N.

Consider Eq. (1) as a 6×6 matrix with m_0 and m_D representing 3×3 unit matrices, as required by the A_4 symmetry. The soft breaking of A_4 means that M may differ slightly from the unit matrix, so that in the basis under which it is diagonal, $\mathcal{M}_{\nu N}$ is given by

$$\mathcal{M}_{\nu N} = \begin{bmatrix} m_0 & 0 & 0 & m_D & 0 & 0 \\ 0 & m_0 & 0 & 0 & m_D & 0 \\ 0 & 0 & m_0 & 0 & 0 & m_D \\ m_D & 0 & 0 & M_1 & 0 & 0 \\ 0 & m_D & 0 & 0 & M_2 & 0 \\ 0 & 0 & m_D & 0 & 0 & M_3 \end{bmatrix}. \tag{2}$$

The ν_e, ν_μ, ν_τ basis is now rotated into the $\nu'_{1,2,3}$ basis, and we may assume whatever pattern is suitable for explaining the atmospheric and solar neutrino data. To be specific, consider bimaximal mixing, i.e.

$$\begin{bmatrix} \nu'_1 \\ \nu'_2 \\ \nu'_3 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & 1/2 & -1/2 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}. \tag{3}$$

Then the eigenstates of $\mathcal{M}_{\nu N}$ are

$$\nu_i = \nu'_i \cos\theta_i - N_i \sin\theta_i, \quad S_i = \nu'_i \sin\theta_i + N_i \cos\theta_i, \tag{4}$$

where $\sin\theta_i \simeq m_D/M_i$, corresponding to the eigenvalues $m_0 - m_D^2/M_i$ and M_i respectively. Since $M_1 \simeq M_2 \simeq M_3$ is still assumed, we have

$$\Delta m_{ij}^2 = \left(m_0 - \frac{m_D^2}{M_i}\right)^2 - \left(m_0 - \frac{m_D^2}{M_j}\right)^2 \simeq \frac{m_\nu m_D^2}{M^3}\Delta M_{ij}^2, \tag{5}$$

where $M = (M_1 + M_2 + M_3)/3$ and $m_\nu = m_0 - m_D^2/M$.

Consider now the effect of S_i on $\nu_\mu \to \nu_e$ oscillations. The well-known expression for this probability is given by

$$P(\nu_\mu \to \nu_e) = -4\sum_i U_{\mu i}U_{ei}\sum_{j>i} U_{\mu j}U_{ej}\sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right). \tag{6}$$

For the L/E values appropriate to the LSND experiment, Δm_{ij}^2 is effectively zero between ν_i and ν_j. Naively, we might expect the contribution from Δm_{ij}^2 between S_i and ν_j, i.e. $M^2 - m_\nu^2$ to be dominant, but that turns out to be negligible. Specifically,

$$\sum_{j=4,5,6} U_{\mu j}U_{ej} = -\sum_{i=1,2,3} U_{\mu i}U_{ei} = \frac{1}{2\sqrt{2}}(\sin^2\theta_1 - \sin^2\theta_2) \simeq \frac{1}{2\sqrt{2}}\left(\frac{\Delta m_{21}^2}{m_\nu M}\right) \simeq 0, \tag{7}$$

where Eq. (5) has been used. Hence the main contribution to Eq. (6) is actually coming from $i = 4$ and $j = 5$, i.e.

$$P(\nu_\mu \to \nu_e) = \frac{1}{2}\sin^2\theta_1\sin^2\theta_2\sin^2\left(\frac{\Delta M_{12}^2 L}{4E}\right). \tag{8}$$

This means that the neutrino mass difference being probed by the LSND experiment is that between S_1 and S_2, and <u>not</u> that between ν_e or ν_μ and S_i. To understand this interesting new phenomenon, we note that if the M_i's were equal, then ν_e, ν_μ, ν_τ would all be exactly degenerate in mass and there could not be any $\nu_\mu \to \nu_e$ oscillation; thus any such effect must be proportional to the difference in the M_i's and not to the difference between M and m_ν.

To fit the LSND data, we take $\Delta M_{21}^2 \simeq 1$ eV^2 and $\sin^2\theta_1 \simeq \sin^2\theta_2 \simeq 0.06$, so that $\sin^2 2\theta_{eff} \simeq 1.8\times10^{-3}$. In that case, Eq. (5) relates them to Δm_{21}^2 which may be as large as 1.2×10^{-4} eV^2 as indicated by the solar data. We then obtain $m_\nu/M \simeq 2\times10^{-3}$. If we take

$m_\nu \simeq 0.2$ eV, then $M \simeq 0.1$ keV. Taking Δm^2_{32} to be as small as 1.2×10^{-3} eV^2 as indicated by the atmospheric data, we obtain $\Delta M^2_{32} \simeq 10$ eV^2 and $\sin^2 2\theta_{eff} \simeq 3.6 \times 10^{-3}$ for $\nu_\mu \to \nu_\tau$ oscillations in the CHORUS [14] and NOMAD [15] experiments, which are just beyond their exclusion boundaries.

We now give the theoretical details of our model. In addition to A_4, we define 2 global U(1) symmetries: $U(1)_L$ and $U(1)_S$. Under these symmetries, the leptons transform as follows: [12] $(\nu_i, l_i)_L \sim (\underline{3}, 1, 0)$, $l_{1R} \sim (\underline{1}, 1, 0)$, $l_{2R} \sim (\underline{1}', 1, 0)$, $l_{3R} \sim (\underline{1}'', 1, 0)$, $N_{iR} \sim (\underline{3}, 1, 1)$. There are 4 scalar doublets: $\Phi_i = (\phi_i^+, \phi_i^0) \sim (\underline{3}, 0, 0)$, $\eta = (\eta^+, \eta^0) \sim (\underline{1}, 0, 1)$, and 1 scalar triplet: $\xi = (\xi^{++}, \xi^+, \xi^0) \sim (\underline{1}, -2, 0)$. The soft terms $\Phi_i^\dagger \eta$ break $U(1)_S$ and A_4, $\xi^\dagger \Phi_i \Phi_j$ break $U(1)_L$ and A_4, $N_i N_j$ break $U(1)_L$, $U(1)_S$, and A_4, hence they may all be naturally small [16]. Note that the smallness of M is protected by <u>both</u> $U(1)_L$ and $U(1)_S$. Assuming m^2_ξ and m^2_η to be positive and large, we then obtain $\langle \xi^0 \rangle$ and $\langle \eta^0 \rangle$ to be small [6, 8] for the terms m_0 and m_D respectively.

Since the 3 sterile neutrinos have masses of order 0.1 keV and their decay lifetimes (through their mixing with the active neutrinos) are much greater than the age of the Universe, they would overclose the Universe unless their relic abundance is greatly reduced. This may be achieved in our scenario because N_{iR} have no gauge interactions and the only Yukawa couplings they have are given by

$$\mathcal{L}_{int} = f \bar{N}_{iR} (\nu_i \eta^0 - l_i \eta^+) + h.c. \tag{9}$$

Hence they decouple from the standard-model particles at the scale M_η which we take to be 1 TeV, and whereas $\nu_{e,\mu,\tau}$ are heated by the subsequent annihilations of nonrelativistic particles, N_{iR} are not [17]. Thus the number densities of the latter are greatly suppressed until the onset of electroweak symmetry breaking. However, the mixing of N_{iR} with ν_i is then large enough to produce the former through active-sterile neutrino oscillations and we are faced again with the abundance problem, together with the nucleosynthesis bound of the effective number of neutrinos n_ν which is restricted to be less than 4 [18]. To evade these cosmological problems, we need N_{iR} to decay quickly as proposed previously [19], but at the expense of the unnatural fine tuning of parameters.

In conclusion, we have constructed in this paper a specific model of 3 active (doublet) and 3 sterile (singlet) neutrinos, which are separately threefold degenerate in mass approximately. We find the new and interesting result that neutrino oscillations between active species are governed by the mass differences among the 3 lighter neutrinos and the parallel mass differences [see Eq. (5)] among the 3 heavier neutrinos, but not the mass difference between the two groups. If bimaximal mixing is assumed for explaining the atmospheric and solar neutrino oscillations (using the matter-enhanced solution for the latter), then a possible explanation of the LSND data is the nonnegligible mixing ($\sin^2\theta \simeq 0.06$) between active and sterile neutrinos. This would then imply sterile neutrino masses of order 0.1 keV (if active neutrino masses are around 0.2 eV) and $\Delta m^2 \simeq 10$ eV^2 for the CHORUS and NOMAD experiments with $\sin^2\theta_{eff} \simeq 3.6 \times 10^{-3}$, which is just barely consistent with their exclusion limits. New data from future long-baseline and medium-baseline neutrino-oscillation experiments will be decisive in confronting this possible 3+3 scenario.

This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837. G.R. also thanks the UCR Physics Department for hospitality and acknowledges INSA (new Delhi) for support.

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